X-551-68-367

PREPRINT

A PROGRAMMED MATHEMATICAL MODEL TO SIMULATE THE BENDING OF RADIO WAVES IN ATMOSPHERIC PROPAGATION

B. ROSENBAUM
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GODDARD SPACE FLIGHT CENTER

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GODDARD SPACE FLIGHT CENTER
Mission & Trajectory Analysis Division
Greenbelt, Maryland

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ABSTRACT

The ray path of an RF signal undergoes bending in the interval of atmospheric propagation between a satellite and an earth-based tracking station. Resultant refraction is a perturbation on observed elevation angles and Doppler shifted signals. As a practical tool for the computation of corrections a mathematical model simulating the tracing of the refracted ray path has been programmed for use by the Goddard Space Flight Center computer facilities.

CONTENTS

ILLUSTRATIONS

LIST OF SYMBOLS

c	reciprocal scale height of troposphere (km^{-1}
f	signal frequency (MHz)
h	altitude (km)
h_m	altitude of N_m
$2H$	scale height of the ionosphere
n	index of refraction
N	refractivity
N_e	electron density (m^{-3})
N_m	maximum of the N_e profile
r	radial distance (km)
r_0	earth radius (km)
β	true elevation angle at earth surface
δ	refraction angle at satellite
ϵ	refraction angle at earth surface
θ	elevation angle at satellite
τ	total bending angle

A PROGRAMMED MATHEMATICAL MODEL TO SIMULATE THE BENDING OF RADIO WAVES IN ATMOSPHERIC PROPAGATION

I. INTRODUCTION

The ray path of an RF signal undergoes bending in the interval of atmospheric propagation from a satellite to an earth-based tracking station. The refraction introduces a perturbation on tracking measurement of elevation angle and measurement of Doppler shift and its conversion to range rate. Correction of the effect can be performed through a calculation of the finite refraction angles between the line-of-sight and the ray path at both the emitter and receiver.

This document presents the mathematical model and its associated computer program to simulate the ray tracing of a signal path in a spherically stratified atmosphere. While many such routines performing similar traces exist (Reference 1), they are not adapted to the newly installed IBM 360 computers at Goddard Space Flight Center (GSFC). This program has been checked out and run at GSFC on computer models IBM 360/91 and IBM 360/95. An exponential model is used for the tropospheric refractivity profile and the Chapman function is used for the ionospheric profile. The refraction effect on elevation angle attributable to the troposphere and ionosphere can thus be calculated from a single routine. The computation procedure is essentially that of the National Bureau of Standards tropospheric ray tracing program (Reference 2). A sample calculation is included in Chapter IV, Users Guide.

II. REFRACTIVITY MODEL

The following equations were used in the construction of the refractivity model.

1. Relation of Refractivity to the Index of Refraction

The relationship of refractivity, N, to the index of refraction, n, is expressed by the equation:

$$N = (n - 1) \times 10^6$$

2. Tropospheric Refractivity Profile

The exponential tropospheric refractivity profile is expressed as:

$$N(r) = N(r_0 + h) = N_0 e^{-ch}$$

where

$$0 \leq h \leq 50 \text{ km}$$

3. Ionospheric Refractivity Profile — Chapman Model

The refractivity in the ionosphere is a function of free electron density, N_e, and signal frequency, f, as expressed in the following equation:

$$N(r) = \frac{-40.38 \times 10^6 N_e (r_0 + h)}{f^2}$$

where

$$h > 50 \text{ km}$$

The altitude profile for electron density is given by the Chapman function:

$$N_e = N_m \exp 1/2(1 - z - e^{-z})$$

where

$$z = \frac{(h - h_m)}{H}$$

III. METHOD OF COMPUTATION

The ray tracing procedure divides the atmosphere into a series of thin laminations (Figure 1). The refraction between consecutive layers is then followed. Snell's law, which for a spherically stratified medium is given by:

$$n_i r_i \cos \theta_i = n_0 r_0 \cos \theta_0,$$

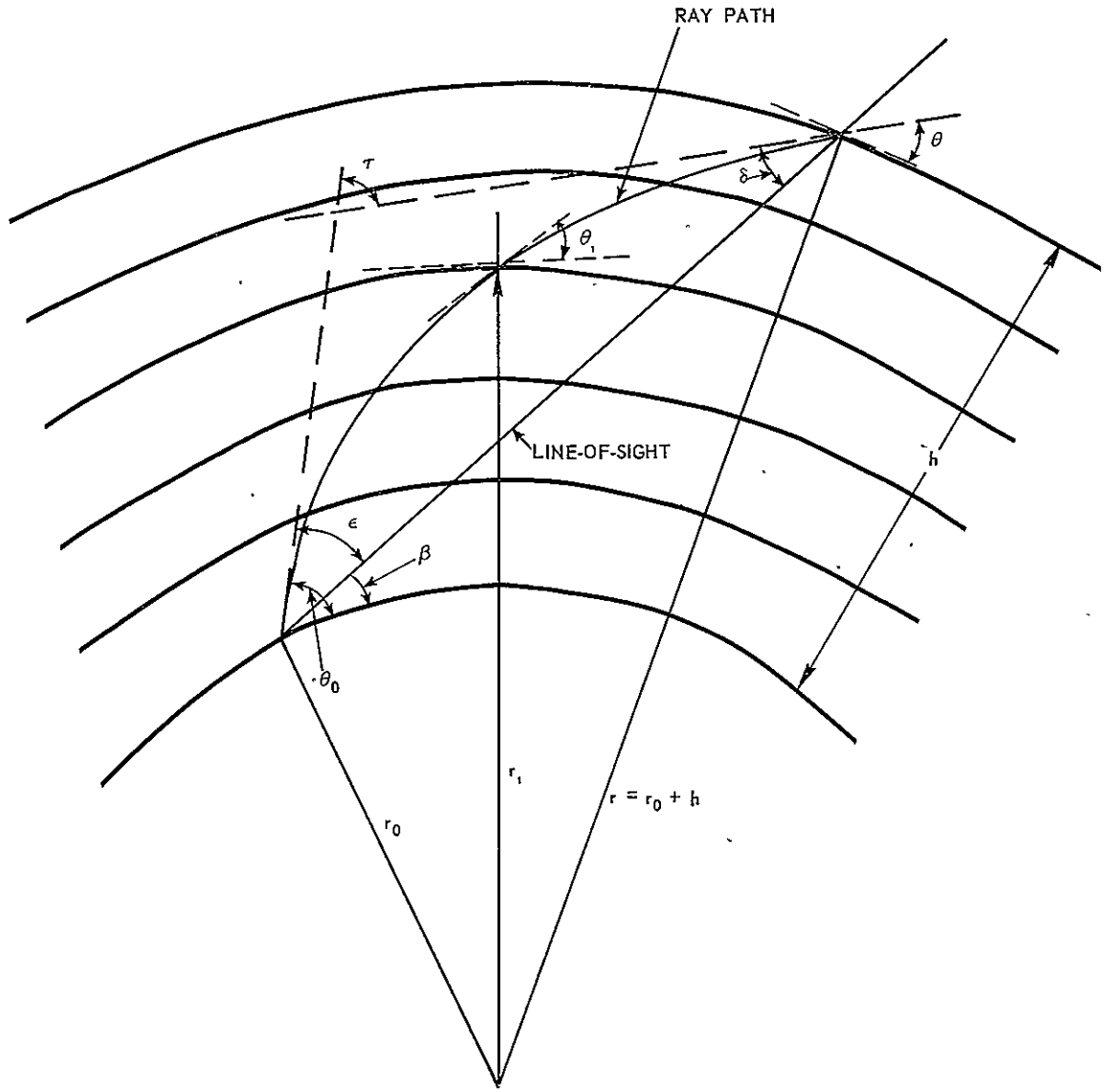


Figure 1. The Geometry of the Ray-Tracing

is the basis for the calculation. This is rewritten for computation purpose as:

$$\sin^2 (\theta_i/2) = \frac{r_0}{2r_i}\left\{ 2 \sin^2 (\theta_0/2) + \frac{r_i - r_0}{r_0} - \frac{(N_0 - N_i)}{n_i} \times 10^{-6} \cos \theta_0 \right\}$$

The incremental bending angle, $\Delta\tau$, is determined by the general formula:

$$\Delta\tau = -\frac{\Delta n}{n} \cos \theta$$

Between the i^{th} and $(i + 1)^{th}$ lamination the bending, $\Delta\tau_i$, is calculated by the expression:

$$\Delta\tau_i = \frac{(N_i - N_{i+1})}{(n_i + n_{i+1})/2} \cot \left((\theta_i + \theta_{i+1})/2\right)$$

The total bending is given by:

$$\tau_i = \sum_{j=1}^{i} \Delta\tau_j$$

The error in the computation of τ_i is made small by making the laminations sufficiently thin.

The refraction angle is calculated from the formula:

$$\tan \epsilon_i = \frac{\cos \tau_i - \sin \tau_i \tan \theta_i - n_i/n_0}{(n_i/n_0) \tan \theta_0 - \sin \tau_i - \cos \tau_i \tan \theta_i}$$

and the refraction angle δ_i can be calculated by the formula:

$$\tan \delta_i = \frac{n_0/n_i - \cos \tau_i - \sin \tau_i \tan \theta_0}{\sin \tau_i - \cos \tau_i \tan \theta_0 + (n_0/n_i) \tan \theta_i}$$

The relation, $\tau_i = \epsilon_i + \delta_i$, can be used as a check on the computation.

IV. USERS GUIDE

1. Introduction

This program computes the bending angle of radio waves in atmospheric propagation.

2. Programming System

The program is written in double precision, FORTRAN IV compiler language. It is operational on the IBM 360/Model 91 and Model 95 computers.

3. Required Input/Output Units

No special equipment configurations are required. Input is a source card deck and data cards. Output is a standard printout.

4. Restrictions

There are no restrictions on this program.

5. Input

The following values are read into the computer in a D10.2 format.

(1) The reciprocal scale height of the troposphere (c),

(2) The maximum electron density (N_m),

(3) The initial value of the refractivity (N_0),

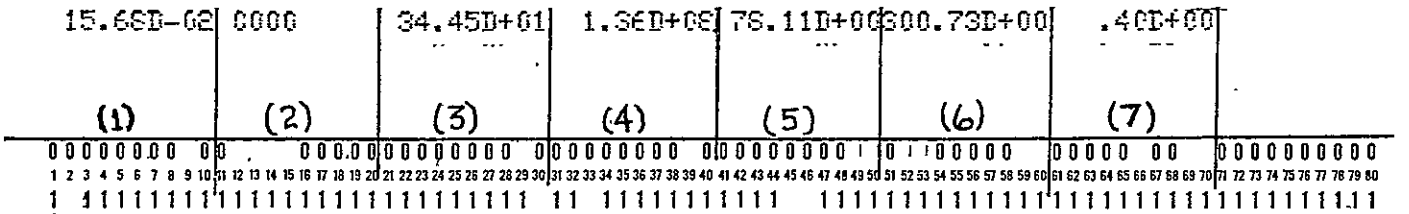
(4) The signal frequency in megahertz (f),

(5) The scale height of the ionosphere in kilometers (2H),

(6) The height of N_m, (h_m), and

(7) The initial value of the angle between the ray path and the radius vector, in radians (θ_0).

A sample input data card is shown below.



The following Job Control Language cards are required to successfully execute the program.

```
C.C.*
1
//DSNAS033bJOBb(G7001H1310,T,DA0023,001005),608,MSGLEVEL=1
//bEXECbFORTRAN
//SOURCE.SYSINbDDb*
```

SOURCE LANGUAGE PROGRAM

```
C.C.*
1
/*
//bEXECbLINKGO
//GO.DATA5bDDb*
```

DATA CARDS

```
C.C.*
1
/*
```

* C.C. = Card Column
b = blank column

6. Output

The values of the following parameters are printed as the input value of H is incremented by the program to its predetermined maximum (3000 km). Format is indicated following the parameter.

H	(altitude in kilometers)	F 7.1
N	(refractivity)	E15.5
$N_i - N_{i+1}$	(difference between successive values of N)	E15.5
θ	(angle between ray path and radius vector, in milliradians)	E15.5
$\Delta\tau$	(discrete portion of the bending angle, in milliradians)	E15.5
τ	(bending angle, in milliradians)	E15.5
ϵ	(refraction angle at the earth surface, in milliradians)	E15.5
δ	(refraction angle at the satellite, in milliradians)	E15.5
$\epsilon + \delta$	(the sum of epsilon and delta, in milliradians)	E15.5

In addition, initial values and constants used in the equations are printed. A sample output follows.

BENDING OF RADIO WAVES IN ATMOSPHERIC PROPAGATION

NM = 0.424850 12 HM = 300.73 H = 78.11 NO = 344.50
INITIAL THETA = 400.00 C = 0.1568 F = 0.136000 09 RO = 6373.015

ALL ANGULAR MEASUREMENTS ARE IN MILLIRADIANS

H	N	N(I) - N(I+1)	THETA	DELTA TAU	TAU	EPSILON	DELTA	EPSILON+DELTA
0.0	0.34450E 03	0.53596E 01	0.40000E 03	0.0	0.0	0.0	0.0	0.0
0.1	0.33914E 03	0.10470E 02	0.40002E 03	0.12672E-01	0.12672E-01	0.63360E-02	0.63359E-02	0.12672E-01
0.3	0.32867E 03	0.10147E 02	0.40007E 03	0.24753E-01	0.37425E-01	0.18811E-01	0.18814E-01	0.37425E-01
0.5	0.31852E 03	0.98339E 01	0.40012E 03	0.23996E-01	0.61411E-01	0.31053E-01	0.30358E-01	0.61411E-01
0.7	0.30869E 03	0.14184E 02	0.40018E 03	0.23242E-01	0.84653E-01	0.43045E-01	0.41608E-01	0.84653E-01
1.0	0.29450E 03	0.90923E 01	0.40025E 03	0.33519E-01	0.11817E 00	0.60552E-01	0.57621E-01	0.11817E 00
1.2	0.28541E 03	0.88116E 01	0.40031E 03	0.21482E-01	0.13966E 00	0.71942E-01	0.67713E-01	0.13966E 00
1.4	0.27660E 03	0.85396E 01	0.40036E 03	0.20816E-01	0.16047E 00	0.83098E-01	0.77373E-01	0.16047E 00
1.6	0.26806E 03	0.82759E 01	0.40041E 03	0.20171E-01	0.18064E 00	0.94026E-01	0.86617E-01	0.18064E 00
1.8	0.25978E 03	0.80204E 01	0.40047E 03	0.19545E-01	0.20019E 00	0.10473E 00	0.95457E-01	0.20019E 00
2.0	0.25176E 03	0.77728E 01	0.40052E 03	0.18939E-01	0.21913E 00	0.11522E 00	0.10391E 00	0.21913E 00
2.2	0.24399E 03	0.75328E 01	0.40059E 03	0.18352E-01	0.23748E 00	0.12549E 00	0.11199E 00	0.23748E 00
2.4	0.23646E 03	0.73003E 01	0.40063E 03	0.17783E-01	0.25526E 00	0.13556E 00	0.11970E 00	0.25526E 00
2.6	0.22916E 03	0.70749E 01	0.40069E 03	0.17231E-01	0.27249E 00	0.14542E 00	0.12707E 00	0.27249E 00
2.8	0.22208E 03	0.68565E 01	0.40075E 03	0.16696E-01	0.28919E 00	0.15503E 00	0.13410E 00	0.28919E 00
3.0	0.21523E 03	0.66448E 01	0.40081E 03	0.16178E-01	0.30537E 00	0.16456E 00	0.14081E 00	0.30537E 00
3.2	0.20858E 03	0.64396E 01	0.40086E 03	0.15677E-01	0.32104E 00	0.17384E 00	0.14721E 00	0.32104E 00
3.4	0.20214E 03	0.62408E 01	0.40092E 03	0.15190E-01	0.33623E 00	0.18293E 00	0.15330E 00	0.33623E 00
3.6	0.19590E 03	0.60482E 01	0.40098E 03	0.14719E-01	0.35095E 00	0.19185E 00	0.15910E 00	0.35095E 00
3.8	0.18985E 03	0.58614E 01	0.40104E 03	0.14262E-01	0.36521E 00	0.20059E 00	0.16463E 00	0.36521E 00
4.0	0.18399E 03	0.13874E 02	0.40110E 03	0.13820E-01	0.37903E 00	0.20915E 00	0.16988E 00	0.37903E 00
4.5	0.17012E 03	0.12828E 02	0.40125E 03	0.32702E-01	0.41174E 00	0.22982E 00	0.18192E 00	0.41174E 00
5.0	0.15729E 03	0.11861E 02	0.40141E 03	0.30223E-01	0.44196E 00	0.24949E 00	0.19247E 00	0.44196E 00
5.5	0.14543E 03	0.10966E 02	0.40157E 03	0.27933E-01	0.46989E 00	0.26822E 00	0.20167E 00	0.46989E 00
6.0	0.13446E 03	0.10139E 02	0.40172E 03	0.25815E-01	0.49571E 00	0.28607E 00	0.20964E 00	0.49571E 00
6.5	0.12432E 03	0.93747E 01	0.40189E 03	0.23858E-01	0.51957E 00	0.30308E 00	0.21649E 00	0.51957E 00
7.0	0.11495E 03	0.86678E 01	0.40205E 03	0.22050E-01	0.54162E 00	0.31929E 00	0.22232E 00	0.54162E 00
7.5	0.10628E 03	0.80142E 01	0.40221E 03	0.20378E-01	0.56199E 00	0.33475E 00	0.22724E 00	0.56199E 00
8.0	0.98268E 02	0.74099E 01	0.40238E 03	0.18833E-01	0.58083E 00	0.34950E 00	0.23133E 00	0.58083E 00
8.5	0.90858E 02	0.68512E 01	0.40254E 03	0.17405E-01	0.59823E 00	0.36358E 00	0.23465E 00	0.59823E 00
9.0	0.84006E 02	0.63345E 01	0.40271E 03	0.16085E-01	0.61432E 00	0.37702E 00	0.23730E 00	0.61432E 00
9.5	0.77672E 02	0.58569E 01	0.40288E 03	0.14865E-01	0.62918E 00	0.38986E 00	0.23932E 00	0.62918E 00
10.0	0.71815E 02	0.10422E 02	0.40305E 03	0.13738E-01	0.64292E 00	0.40212E 00	0.24080E 00	0.64292E 00
11.0	0.61393E 02	0.89096E 01	0.40339E 03	0.24429E-01	0.66735E 00	0.42504E 00	0.24231E 00	0.66735E 00
12.0	0.52483E 02	0.76166E 01	0.40374E 03	0.20864E-01	0.68821E 00	0.44601E 00	0.24220E 00	0.68821E 00
13.0	0.44867E 02	0.65113E 01	0.40409E 03	0.17819E-01	0.70603E 00	0.46524E 00	0.24079E 00	0.70603E 00
14.0	0.38355E 02	0.55663E 01	0.40444E 03	0.15219E-01	0.72125E 00	0.48289E 00	0.23836E 00	0.72125E 00
15.0	0.32789E 02	0.47618E 02	0.40479E 03	0.12998E-01	0.73425E 00	0.49913E 00	0.23512E 00	0.73425E 00
20.0	0.14971E 02	0.11850E 02	0.40657E 03	0.41485E-01	0.77573E 00	0.56271E 00	0.21302E 00	0.77573E 00
30.0	0.31208E 01	0.24702E 01	0.41016E 03	0.27346E-01	0.80312E 00	0.63767E 00	0.16545E 00	0.80312E 00
40.0	0.65057E 00	0.51495E 00	0.41372E 03	0.56535E-02	0.80877E 00	0.67923E 00	0.12955E 00	0.80877E 00
50.0	0.13562E 00	0.26738E 00	0.41725E 03	0.11672E-02	0.80994E 00	0.70483E 00	0.10511E 00	0.80994E 00
60.0	-0.13176E 00	0.32622E 00	0.42076E 03	0.60035E-03	0.81054E 00	0.72205E 00	0.88495E-01	0.81054E 00

H	N	N(I) - N(I+1)	THETA	DELTA TAU	TAU	EPSILON	DELTA	EPSILON+DELTA
70.0	-0.45798E 00	0.902C7F 00	0.42423F 03	0.72568F-03	0.31127E 00	0.734446F 00	0.76823F-01	0.81127F 00
80.0	-0.13600E 01	0.21565F 01	0.42762E 03	0.19944F-02	0.81326F 00	0.74399F 00	0.69356F-01	0.81326F 00
90.0	-0.35166E 01	0.45328E 01	0.43099E 03	0.47108F-02	0.81797E 00	0.75142F 00	0.66344F-01	0.81797F 00
100.0	-0.80494E 01	0.85026E 01	0.43433E 03	0.98146E-02	0.82778E 00	0.75850E 00	0.69277F-01	0.82778F 00
110.0	-0.16552E 02	0.14420E 02	0.43763F 03	0.18251F-01	0.84603F 00	0.765365 00	0.80669F-01	0.84603F 00
120.0	-0.30972F 02	0.22368E 02	0.44088F 03	0.30691E-01	0.87672F 00	0.77304E 00	0.10368F 00	0.87672F 00
130.0	-0.53340E 02	0.32254E 02	0.44409E 03	0.47211F-01	0.92393F 00	0.78240F 00	0.14153F 00	0.92393F 00
140.0	-0.85394E 02	0.42809E 02	0.44724F 03	0.67104F-01	0.99104F 00	0.79433F 00	0.19671E 00	0.99104F 00
150.0	-0.12820E 03	0.53693F 02	0.45032F 03	0.88909F-01	0.10799F 01	0.80361E 00	0.27034E 00	0.10799F 01
160.0	-0.18190E 03	0.63660E 02	0.45337E 03	0.11055E 00	0.11906E 01	0.82890E 00	0.35169E 00	0.11906F 01
170.0	-0.24556F 03	0.71748F 02	0.45637E 03	0.13019E 00	0.13208E 01	0.85263E 00	0.46815E 00	0.13208F 01
180.0	-0.31730E 03	0.77228E 02	0.45932F 03	0.14564F 00	0.14664F 01	0.88006E 00	0.58547E 00	0.14664F 01
190.0	-0.39453E 03	0.79688E 02	0.46224F 03	0.15563E 00	0.16221E 01	0.91378E 00	0.70927E 00	0.16221F 01
200.0	-0.47422E 03	0.79048F 02	0.46512F 03	0.15643F 00	0.17815F 01	0.95073E 00	0.83076E 00	0.17815F 01
210.0	-0.55327E 03	0.75521E 02	0.46799F 03	0.15704F 00	0.19385F 01	0.99123F 00	0.94729F 00	0.19385F 01
220.0	-0.62879E 03	0.69533F 02	0.47083E 03	0.14898F 00	0.20875E 01	0.10346F 01	0.10529F 01	0.20875E 01
230.0	-0.69832E 03	0.61640F 02	0.47366F 03	0.13622F 00	0.22237F 01	0.10799E 01	0.11438F 01	0.22237F 01
240.0	-0.75996F 03	0.52444E 02	0.47648E 03	0.11993F 00	0.23436F 01	0.11265E 01	0.12171F 01	0.23436F 01
250.0	-0.81241E 03	0.42525E 02	0.47930F 03	0.10134E 00	0.24450F 01	0.11735F 01	0.12715F 01	0.24450F 01
260.0	-0.85493E 03	0.32400F 02	0.48211E 03	0.81611F-01	0.25266F 01	0.12201E 01	0.13065F 01	0.25266F 01
270.0	-0.88733E 03	0.22495F 02	0.48491E 03	0.61758E-01	0.25884F 01	0.12658E 01	0.13226F 01	0.25884F 01
280.0	-0.90983E 03	0.13135F 02	0.48772E 03	0.42588E-01	0.26309E 01	0.13099E 01	0.13210F 01	0.26309F 01
290.0	-0.92296E 03	0.45516E 01	0.49051E 03	0.24702F-01	0.26556F 01	0.13521E 01	0.13035F 01	0.26556F 01
300.0	-0.92751E 03	-0.31149F 01	0.49330F 03	0.85021F-02	0.26641F 01	0.13920F 01	0.12721F 01	0.26641F 01
310.0	-0.92440E 03	-0.97944F 01	0.49609E 03	-0.57797F-02	0.26584F 01	0.14294E 01	0.12289F 01	0.26584F 01
320.0	-0.91460E 03	-0.15475E 02	0.49886E 03	-0.18053F-01	0.26403F 01	0.14642F 01	0.11761E 01	0.26403F 01
330.0	-0.89913E 03	-0.20186F 02	0.50162E 03	-0.28336F-01	0.26120E 01	0.14962E 01	0.11158F 01	0.26120F 01
340.0	-0.87894E 03	-0.23987E 02	0.50436E 03	-0.36721F-01	0.25753E 01	0.15255E 01	0.10498F 01	0.25753F 01
350.0	-0.85496E 03	-0.26956F 02	0.50710E 03	-0.43353E-01	0.25319E 01	0.15521E 01	0.97984F 00	0.25719F 01
360.0	-0.82800E 03	-0.29181F 02	0.50981E 03	-0.48407F-01	0.24835E 01	0.15760E 01	0.90745F 00	0.24835F 01
370.0	-0.79882E 03	-0.30754F 02	0.51251E 03	-0.52070F-01	0.24314E 01	0.15975E 01	0.83391F 00	0.24314F 01
380.0	-0.76806E 03	-0.31763F 02	0.51519E 03	-0.54530E-01	0.23769E 01	0.16166E 01	0.75028E 00	0.23769F 01
390.0	-0.73630E 03	-0.32295E 02	0.51785F 03	-0.55968E-01	0.23209E 01	0.16335E 01	0.68744E 00	0.23209F 01
400.0	-0.70401E 03	-0.32426E 02	0.52049E 03	-0.56553F-01	0.22644F 01	0.16482E 01	0.61614E 00	0.22644F 01
410.0	-0.67158E 03	-0.32229E 02	0.52311E 03	-0.56436F-01	0.22079E 01	0.16611E 01	0.54688E 00	0.22079F 01
420.0	-0.63935E 03	-0.31766F 02	0.52572E 03	-0.55754F-01	0.21522E 01	0.16721E 01	0.48010E 00	0.21522F 01
430.0	-0.60759E 03	-0.31093E 02	0.52830E 03	-0.54624F-01	0.20976E 01	0.16815E 01	0.41608E 00	0.20976F 01
440.0	-0.57649E 03	-0.30256F 02	0.53086E 03	-0.53149F-01	0.20444E 01	0.16893E 01	0.35507E 00	0.20444F 01
450.0	-0.54624E 03	-0.29296F 02	0.53340E 03	-0.51416E-01	0.19930E 01	0.16958E 01	0.29716E 00	0.19930F 01
460.0	-0.51694E 03	-0.28247E 02	0.53590E 03	-0.49496F-01	0.19435F 01	0.17011F 01	0.24249E 00	0.19435F 01
470.0	-0.48869E 03	-0.27137F 02	0.53843F 03	-0.47450F-01	0.18961E 01	0.17052F 01	0.19085E 00	0.18961F 01
480.0	-0.46156E 03	-0.25991E 02	0.54091F 03	-0.45327E-01	0.18507E 01	0.17083E 01	0.14243E 00	0.18507F 01
490.0	-0.43557E 03	-0.24826E 02	0.54338E 03	-0.43168F-01	0.18076E 01	0.17105E 01	0.97089E-01	0.18076F 01
500.0	-0.41074E 03	-0.23659E 02	0.54582E 03	-0.41005E-01	0.17666E 01	0.17119E 01	0.54736E-01	0.17666F 01
510.0	-0.38708F 03	-0.22501E 02	0.54825E 03	-0.38862F-01	0.17277F 01	0.17124E 01	0.15266E-01	0.17277E 01
520.0	-0.36458E 03	-0.21364F 02	0.55066E 03	-0.36760E-01	0.16909E 01	0.17124E 01	-0.21441E-01	0.16909F 01
530.0	-0.34322F 03	-0.20252F 02	0.55305E 03	-0.34713E-01	0.16562E 01	0.17117E 01	-0.55519E-01	0.16562F 01
540.0	-0.32296E 03	-0.19174F 02	0.55542F 03	-0.32732F-01	0.16235E 01	0.17106E 01	-0.87080E-01	0.16235F 01
550.0	-0.30379E 03	-0.18132E 02	0.55777E 03	-0.30825E-01	0.15927E 01	0.17089E 01	-0.11629E 00	0.15927F 01
560.0	-0.28566E 03	-0.17130F 02	0.56011E 03	-0.28998E-01	0.15637E 01	0.17069E 01	-0.14324E 00	0.15637E 01
570.0	-0.26853E 03	-0.16168E 02	0.56243F 03	-0.27253F-01	0.15364F 01	0.17045E 01	-0.16809E 00	0.15364F 01
580.0	-0.25236E 03	-0.15249F 02	0.56473F 03	-0.25592F-01	0.15108F 01	0.17019E 01	-0.19097E 00	0.15108F 01
590.0	-0.23711E 03	-0.14373E 02	0.56702F 03	-0.24014F-01	0.14869E 01	0.16999E 01	-0.21200E 00	0.14869F 01

H	N	N(I) - N(I+1)	THETA	DELTA TAU	TAU	EPSILON	DELTA	EPSILON+DELTA
600.0	-0.22274E 03	-0.13539E 02	0.56929E 03	-0.22552E-01	0.14643E 01	0.16956E 01	-0.23129E 00	0.14643E 01
610.0	-0.20920E 03	-0.12747E 02	0.57154E 03	-0.21138E-01	0.14432E 01	0.16921E 01	-0.24986E 00	0.14432E 01
620.0	-0.19648E 03	-0.11995E 02	0.57378E 03	-0.19774E-01	0.14234E 01	0.16885E 01	-0.26513E 00	0.14234E 01
630.0	-0.18446E 03	-0.11284E 02	0.57601E 03	-0.18518E-01	0.14049E 01	0.16848E 01	-0.27988E 00	0.14049E 01
640.0	-0.17317E 03	-0.10611E 02	0.57822E 03	-0.17335E-01	0.13875E 01	0.16809E 01	-0.29332E 00	0.13875E 01
650.0	-0.16256E 03	-0.99761E 01	0.58041E 03	-0.16223E-01	0.13713E 01	0.16769E 01	-0.30656E 00	0.13713E 01
660.0	-0.15259E 03	-0.93748E 01	0.58259E 03	-0.15178E-01	0.13561E 01	0.16728E 01	-0.31664E 00	0.13561E 01
670.0	-0.14321E 03	-0.88086E 01	0.58476E 03	-0.14197E-01	0.13419E 01	0.16686E 01	-0.32668E 00	0.13419E 01
680.0	-0.13440E 03	-0.82749E 01	0.58691E 03	-0.13277E-01	0.13287E 01	0.16644E 01	-0.33575E 00	0.13287E 01
690.0	-0.12613E 03	-0.77721E 01	0.58904E 03	-0.12415E-01	0.13163E 01	0.16602E 01	-0.34391E 00	0.13163E 01
700.0	-0.11836E 03	-0.72986E 01	0.59117E 03	-0.11607E-01	0.13046E 01	0.16559E 01	-0.35124E 00	0.13046E 01
710.0	-0.11106E 03	-0.13287E 02	0.59328E 03	-0.10850E-01	0.12938E 01	0.16516E 01	-0.35779E 00	0.12938E 01
730.0	-0.97770E 02	-0.11709E 02	0.59746E 03	-0.19618E-01	0.12742E 01	0.16431E 01	-0.36879E 00	0.12742E 01
750.0	-0.86061E 02	-0.10315E 02	0.60160E 03	-0.17134E-01	0.12570E 01	0.16344E 01	-0.37773E 00	0.12570E 01
770.0	-0.75747E 02	-0.90838E 01	0.60567E 03	-0.14961E-01	0.12421E 01	0.16256E 01	-0.38378E 00	0.12421E 01
790.0	-0.66663E 02	-0.79961E 01	0.60971E 03	-0.13062E-01	0.12290E 01	0.16176E 01	-0.38844E 00	0.12290E 01
810.0	-0.58665E 02	-0.70410E 01	0.61369E 03	-0.11403E-01	0.12176E 01	0.16092E 01	-0.39158E 00	0.12176E 01
830.0	-0.51624E 02	-0.61977E 01	0.61763E 03	-0.99542E-02	0.12077E 01	0.16011E 01	-0.39344E 00	0.12077E 01
850.0	-0.45426E 02	-0.54548E 01	0.62153E 03	-0.86894E-02	0.11990E 01	0.15932E 01	-0.39429E 00	0.11990E 01
870.0	-0.39971E 02	-0.48005E 01	0.62539E 03	-0.75854E-02	0.11914E 01	0.15855E 01	-0.39409E 00	0.11914E 01
890.0	-0.35171E 02	-0.42245E 01	0.62919E 03	-0.66220E-02	0.11848E 01	0.15779E 01	-0.39318E 00	0.11848E 01
910.0	-0.30946E 02	-0.37175E 01	0.63296E 03	-0.57812E-02	0.11790E 01	0.15706E 01	-0.39163E 00	0.11790E 01
930.0	-0.27229E 02	-0.32712E 01	0.63669E 03	-0.50474E-02	0.11739E 01	0.15635E 01	-0.38956E 00	0.11739E 01
950.0	-0.23957E 02	-0.28787E 01	0.64038E 03	-0.44071E-02	0.11695E 01	0.15566E 01	-0.38705E 00	0.11695E 01
970.0	-0.21079E 02	-0.25326E 01	0.64403E 03	-0.38483E-02	0.11657E 01	0.15499E 01	-0.38418E 00	0.11657E 01
990.0	-0.18546E 02	-0.11409E 01	0.64764E 03	-0.33605E-02	0.11623E 01	0.15434E 01	-0.38103E 00	0.11623E 01
1000.0	-0.17397E 02	-0.17368E 02	0.64942E 03	-0.15172E-02	0.11608E 01	0.15402E 01	-0.37936E 00	0.11608E 01
2000.0	-0.28872E-01	-0.28824E-01	0.79368E 03	-0.19742E-01	0.11411E 01	0.13717E 01	-0.23060E 00	0.11411E 01
3000.0	-0.47913E-04	-0.47913E-04	0.89377E 03	-0.25546E-04	0.11413E 01	0.13078E 01	-0.16681E 00	0.11410E 01

7. Program Listing

A complete listing of the program follows:

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G LEVEL 1, MOD 1              MAIN              DATE = 68128          19/59/32

      DOUBLE PRECISION THETA(125),DELTAU(125),EPS(125),SN(125),AN(125),
     1HAUX(6)  ,TAU(125),GAMMA(125),H(125)
      DOUBLE PRECISION DARSIN,DEXP,DSQRT,DATAN,DTAN,DCOS,THETA0
      DOUBLE PRECISION COTHE,TATHE
      DOUBLE PRECISION D,E,SUM,C,GAM1,EPA,A,B,DEN1,DEN2,DEN3,ANUM1,
     1ANUM2,ANUM3,SN0
      DOUBLE PRECISION ANM,HM,HH,AN0,F,R0,ANE,Z,EX1,EX2,ARG,DIFF
      EQUIVALENCE (HAUX(1),H(1))
      DATA HAUX/0.D0,.1D0,.3D0,.5D0,.7D0,1.0D0/
  100 FORMAT (8D10.2)
      DO 40 I=7,21
   40 H(I)=H(I-1)+.2D0
      DO 42 I=22,33
   42 H(I)=H(I-1)+.5D0
      DO 43 I=34,38
   43 H(I)=H(I-1) + 1.0
      H(39)=20.0D0
      DO 44 I=40,108
   44 H(I)=H(I-1)+10.0D0
      DO 45 I=109,122
   45 H(I)=H(I-1)+20.0D0
      H(123)=1000.0D0
      H(124)=2000.0D0
      H(125)=3000.0D0
C     PROGRAM STOPS WHEN THERE ARE NO MORE DATA CARDS TO BE READ IN
  200 READ(5,100,END=122) C,ANM,AN0,F,HH,HM,THETA0
      ANM=4.24855D+11
      TATHE=DTAN(THETA0)
      COTHE=DCOS(THETA0)
      T0=THETA0*1.D+3
      R0=6373.015D0
      SN0=1.D0+AN0*1.D-6
      EX1=2.D0*(DSIN(THETA0/2.D0))**2
      DO 62 I=1,125
      AH=H(I)
      IF(AH.GT.50.0)  GO TO 58
      AN(I)=AN0*DEXP(-C*H(I))
      SN(I)=1.D0+AN(I)*1.D-6
      GO TO 60
   58 Z=(H(I)-HM)/HH
      ANE=ANM*DEXP((1.D0-Z-DEXP(-Z))/2.D0)
      AN(I)=(-40.38D0*ANE*1.D+6)/F**2
      SN(I)=1.D0 + AN(I)*1.D-6
   60 CONTINUE
      EX2=(AN0-AN(I)) /SN(I) *1.D-6*COTHE
      ARG=DSQRT(R0/(2.*(R0+H(I))) * (EX1 +H(I)/R0 -EX2))
      THETA(I)=2.D0*DARSIN(ARG)
   62 CONTINUE
      WRITE(6,105) ANM,HM,HH,AN0,T0,C,F,R0
      WRITE(6,108)
      WRITE(6,110)
      ILCNT=14
      THETA(1)=THETA0
      EPS(1)=0.0D0
      GAMMA(1)=0.0D0
      TAU(1)=0.0D0
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V. REFERENCES

1. Bjork, C. A. and W. M. Layson, Comparison of Refraction Correction Techniques, PAA Technical Memo No. 65, ETV-TM-67-2 (1967).

2. Bean, B. R., and G. D. Thayer, CRPL Exponential Reference Atmosphere, U. S. Department of Commerce, National Bureau of Standards (now ESSA), October 29, 1959.